Mathematics - Course 421

STANDARD NOTATION

Introduction to Powers of 10

A *power of 10* consists of the *base* 10 raised to some *exponent:*

$$\left.\begin{array}{l} 10^{n} \leftarrow \text{exponent} \\ 10 \leftarrow \text{base} \end{array}\right\} \quad \text{power}$$

10^n stands for n factors of 10. For example,

$$10^5 = 10 \times 10 \times 10 \times 10 \times 10$$

Definitions:

$$\boxed{\begin{array}{l} 10^{-n} = \dfrac{1}{10^n} \\ 10^0 = 1 \end{array}}$$

Thus:

$$\begin{array}{l} \vdots \\ 10^3 = 1000 \\ 10^2 = 100 \\ 10^1 = 10 \\ 10^0 = 1 \\ 10^{-1} = .1 \\ 10^{-2} = .01 \\ 10^{-3} = .001 \\ \vdots \end{array}$$

Powers of 10 are multiplied according to the format,

$$\boxed{10^n \times 10^m = 10^{n+m}},$$

since (n factors of 10) x (m factors of 10) = (m+n) factors of 10.

Powers of 10 are divided according to the format,

$$\boxed{\frac{10^n}{10^m} = 10^{n-m}}$$

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Example 1: $10^5 \times 10^8 = 10^{5+8} = 10^{13}$

Example 2: $10^8 \times 10^{-5} = 10^{8+(-5)} = 10^3$

Example 3: $\dfrac{10^5}{10^8} = 10^{5-8} = 10^{-3}$

Example 4: $\dfrac{10^8}{10^{-5}} = 10^{8-(-5)} = 10^{13}$

Example 5:
$$\frac{10^5 \times 10^{-7} \times 10^3}{10^{-11} \times 10^3} = \frac{10^{5+(-7)+3}}{10^{-11+3}}$$
$$= \frac{10^1}{10^{-8}}$$
$$= 10^{1-(-8)}$$
$$= 10^9$$

Combining Powers of 10 with Decimal Coefficients

A power of 10 can be combined with a decimal *coefficient*,

eg, 4.1×10^6

coefficient (4.1) — power of 10 (10^6)

Recall that shifting the decimal point left one place decreases a number by a factor of 10. Thus the decimal may be shifted left n places in a number if it is multiplied by 10^n to compensate.

eg, $4 = .4 \times 10^1$

$= .04 \times 10^2$

$= .004 \times 10^3$

etc.

Similarly, shifting the decimal point right one place increases a number by a factor of 10. Thus the decimal may be shifted right n places if the number is multiplied by 10^{-n} to compensate.

eg, $4. = 40. \times 10^{-1}$

$= 400. \times 10^{-2}$

$= 4000. \times 10^{-3}$

etc.

Example 1: $5280 = 5.280 \times 10^3$

Example 2: $0.0043 = 4.3 \times 10^{-3}$

Example 3: $65.4 \times 10^2 = 6.54 \times 10^2 \times 10$
$= 6.54 \times 10^3$

(1 move left ⇒ 1 additional factor of 10
⇒ exponent increases by 1)

Example 4: $0.0571 \times 10^{-6} = 5.71 \times 10^{-6} \times 10^{-2}$

$= 5.71 \times 10^{-8}$

(2 moves right ⇒ exponent decreases by 2)

Standard Notation

To express a number in *standard notation (S.N.)* rewrite the number with one nonzero digit left of the decimal point, and multiply by a power of 10 to compensate.

Example 1: Distance travelled by light in one year, ie, one light year is

$9{,}460{,}000{,}000{,}000{,}000 = 9.46 \times 10^{15}$ meters

Example 2: Fission cross section of U^{235} nucleus, for thermal neutrons is

$0.000{,}000{,}000{,}000{,}000{,}000{,}000{,}58 = 5.8 \times 10^{-22}$ cm^2

Example 3: $613 \times 10^4 = 6.13 \times 10^6$

Advantages of Standard Notation

(1) Convenient notation for very large or very small numbers (cf Examples 1 and 2 above), for both ease of writing and ease of comparison.

(2) Facilitates rapid mental calculation.

(3) Shows number of significant figures explicitly, where ambiguity might exist in ordinary decimal notation (cf lesson 421.10-2).

The Four Basic Operations with Numbers in Standard Notation

1. Add numbers in standard notation according to the format,

$$\boxed{a \times 10^n + b \times 10^n = (a + b) \times 10^n}$$

Note that both numbers must have the same power of 10, and that the power of 10 does not change in the addition (similarly for subtraction).

Example 1: $2 \times 10^3 + 3 \times 10^3 = (2 + 3) \times 10^3$

$= 5 \times 10^3$

Example 2: $4.73 \times 10^{-5} + 2.18 \times 10^{-5} = 6.91 \times 10^{-5}$

Example 3: $6.93 \times 10^8 + 4.51 \times 10^6$

$= 6.93 \times 10^8 + .0451 \times 10^8$ (convert to same powers)

$= 6.98 \times 10^8$ (Sum justified to 2 D.P.)

Example 4: $9.78 \times 10^{12} + 5.14 \times 10^{11}$

$= 9.78 \times 10^{12} + .514 \times 10^{12}$ (convert to same powers)

$= 10.29 \times 10^{12}$ (Sum justified to 2 D.P.)

$= 1.029 \times 10^{13}$ (Adjust decimal, power to recover answer in S.N.)

2. Subtract numbers in standard notation according to the format,

$$\boxed{a \times 10^n - b \times 10^n = (a - b) \times 10^n}$$

Example 1: $7 \times 10^5 - 3 \times 10^5 = 4 \times 10^5$

<u>Example 2</u>: $4.65 \times 10^{-8} - 9.24 \times 10^{-10}$

$= 4.65 \times 10^{-8} - 0.0924 \times 10^{-8}$ (convert to same powers)

$= 4.56 \times 10^{-8}$ (difference justified to 2 D.P.)

<u>Example 3</u>: $6.25 \times 10^{12} - 11.3 \times 10^{13}$

$= 0.625 \times 10^{13} - 11.3 \times 10^{13}$ (convert to same powers)

$= -10.7 \times 10^{13}$ (difference justified to 1 D.P.)

$= -1.07 \times 10^{14}$ (adjust decimal, power to recover answer in S.N.)

3. <u>Multiply</u> two numbers in standard notation according to to the format,

$$\boxed{(a \times 10^n)(b \times 10^m) = ab \times 10^{n+m}}$$

<u>Example 1</u>: $2 \times 10^6 \times 3 \times 10^2 = (2 \times 3) \times 10^{6+2}$

$= 6 \times 10^8$

<u>Example 2</u>: $4.7 \times 10^6 \times 6.2 \times 10^{-3}$

$= 29 \times 10^3$ (product justified to 2 S.F.)

$= 2.9 \times 10^4$ (express answer in S.N.)

4. <u>Divide</u> two numbers in standard notation according to the format,

$$\boxed{(a \times 10^n) \div (b \times 10^m) = (a \div b) \times 10^{n-m}}$$

<u>Example 1</u>: $(7 \times 10^6) \div (2 \times 10^{-2}) = (7 \div 2) \times 10^{6-(-2)}$

$= 3.5 \times 10^8$

Example 2: $2.4 \times 10^5 \div 6.9 \times 10^9$

$= 0.35 \times 10^{-4}$ (quotient justified to 2 S.F.)

$= 3.5 \times 10^{-5}$ (express answer in S.N.)

Evaluating Complex Expressions Using Numbers in Standard Notation

(1) Do operations in established order of precedence (cf lesson 421.10-1).

(2) Retain one more D.P. or S.F. than justified in intermediate calculations (to avoid introducing unnecessary 'rounding-off error').

(3) Round off final answer to correct number of digits justified.

Example 1: $2.2 \times 10^2 \div (8.1 \times 10^4) + 1.7 \times 10^{-6} \times 4.6 \times 10^3$

$= 0.272 \times 10^{-2} + 7.82 \times 10^{-3}$ (÷, x precede +; retain 3 S.F. temporarily)

$= 2.72 \times 10^{-3} + 7.82 \times 10^{-3}$ (convert to same power)

$= 10.54 \times 10^{-3}$ (last digit not significant)

$= 1.05 \times 10^{-2}$ (answer in S.N.)

Example 2: Recall that division bar acts as a bracket, requiring evaluation of numerator and denominator prior to division, as follows:

$$\frac{4.7 \times 10^6 + 2.1 \times 10^7}{6.8 \times 10^{11} \times 1.4 \times 10^{-6}}$$

$$= \frac{.47 \times 10^7 + 2.1 \times 10^7}{6.8 \times 1.4 \times 10^{11+(-6)}}$$ (convert to same powers in numerator)

$$= \frac{2.57 \times 10^7}{9.52 \times 10^5}$$ (retain extra digit temporarily)

$= 0.27 \times 10^2$ (answer justified to 2 S.F.)

$= 2.7 \times 10^1$ (answer in S.N.)

ASSIGNMENT

1. Evaluate:

(a) $10^3 \times 10^4 =$ (b) $10^3 \div 10^2 =$

(c) $10^9 \times 10^{-3} =$ (d) $10^9 \div 10^{-3} =$

(e) $10^{-4} \times 10^{-4} =$ (f) $10^{11} \div 10^{20} =$

(g) $10^4 \times 10^{-8} =$ (h) $10^4 \div 10^8 =$

2. Change to a simpler form:

(a) $\frac{1}{10^2} =$ (b) $\frac{1}{10^6 \times 10^3} =$

(c) $\frac{1}{10^{-2}} =$ (d) $\frac{1}{10^{-9} \times 10^9} =$

(e) $-\frac{1}{10^7} =$ (f) $\frac{1}{10^{-13}} =$

(g) $\frac{10^9 \times 10^7}{10^6} =$ (h) $\frac{10^{-17} \times 10^{19}}{10^{20} \times 10^{-5}} =$

(i) $\frac{10^{-11} \times 10^{12}}{10^{-8}} =$ (j) $\frac{10^{21} \times 10^{-19}}{10^3 \times 10^4 \times 10^6} =$

(k) $\frac{10^3}{10^{-12} \times 10^2} =$ (l) $\frac{-10^2 \times 10^3 \times 10^{17}}{10^4 \times 10^{17}} =$

3. Rewrite the following in decimal form:

a) 10^2 b) 10^{-3}

c) 10^5 d) 10^{-6}

e) 10^6 f) 10^{-4}

4. Convert the following to standard notation:

(a) 165 000 (b) .00693

(c) 37.5 (d) .025

(e) 2934 (f) .00101

(g) 10000 (h) .00020

(i) -249 (j) .97

(k) 176×10^{-3} (l) $.0027 \times 10^{3}$

(m) 957×10^{2} (n) $.0175 \times 10^{-12}$

(o) $.024 \times 10^{9}$ (p) $.032 \times 10^{14}$

5. Calculate the following:

(a) $9.3 \times 10^{2} + 1.5 \times 10^{3} =$

(b) $4.6 \times 10^{12} + 9.9 \times 10^{11} =$

(c) $9.4 \times 10^{12} - 1.2 \times 10^{14} =$

(d) $7.5 \times 10^{2} - 5.0 \times 10^{3} =$

(e) $4.5 \times 10^{12} - 4.5 \times 10^{9} =$

6. Express answers in scientific notation:

(a) $3.7 \times 10^{2} \times 2.5 \times 10^{3} =$

(b) $2.5 \times 10^{9} \div 3.6 \times 10^{3} =$

(c) $\dfrac{9.7 \times 10^{12} \times 3.3 \times 10^{10}}{9.5 \times 10^{15}} =$

(d) $\dfrac{3.2 \times 10^{13} \times 2.2 \times 10^{-12}}{1.3 \times 10^{10} \times 9.9 \times 10^{2}} =$

(e) $\dfrac{2.8 \times 10^{-12} \times 1.1 \times 10^{11}}{8.0 \times 10^{3} \times 7.0 \times 10^{-8}} =$

7. Express answers in scientific notation.

(a) $$\frac{7.5 \times 10^{2} + 5.0 \times 10^{3} \times 2.0 \times 10^{-1}}{2.5 \times 10^{2} \times 3.0 \times 10} =$$

(b) $$\frac{(8.6 \times 10^{-14} + 9.9 \times 10^{-13}) \times 2.0 \times 10^{12}}{4.6 \times 10^{3} \times 5.0} =$$

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